Combined Control Strategies for Controlling the Trajectory of a Quadcopter

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Abstract—**The aim of this paper is to show the design of a combined controller, using Backstepping and sliding mode control concepts, for controlling the trajectory following of a quadcopter. The Backstepping-sliding mode controller is compared against a Backstepping- PID controller for circular trajectory. The results by simulations demonstrated the advantages of the proposed approach. The ISE performance index is used to measure the performance and robustness of both controllers.**

Resumen— **El objetivo de este proyecto es mostrar el diseño de un control combinado utilizando los conceptos de Backstepping y Control de Modos Deslizantes para el control de seguimiento de trayectoria de un cuadricóptero. El controlador Backstepping-Modos Deslizantes es comparado con un controlador Backstepping-PID en una trayectoria circular. Los resultados de las simulaciones demuestran las ventajas del enfoque propuesto. El índice de rendimiento ISE se utiliza para medir el desempeño y la robustez de los dos controladores.**

I. Introduction

A quadcopter is an UAV (Unmanned Aerial Vehicle), used in many fields such as ground exploration, data collection and monitoring in areas as diverse as military, research, agriculture, maintenance and security among many more [1].

In order of making those tasks in the best possible way, it is important the development of control mechanisms that seek to ensure the smooth implementation of trajectories planned by the operators.

Due to the complexity of the dynamical system that governs the behaviour of a quadcopter, most of the controller approaches are not able to stabilize properly the position or follow the desired trajectory ([2], [3])

Some works related with this topic have been presented. Arellano-Muro, Carlos Vega, Luis Castillo,B. and Loukianov, Alexander [4], proposed a Backstepping control to solve the trajectory tracking problem and ensures robustness against external forces and parameters variations.

Kacimi, Mokhtari and Kouadri [5], presented in their paper a sliding mode controller to guarantee Lyapunov stability and synthesize tracking errors. It takes into account nonlinearities.

Ramirez, Parra, Sánchez, and Garcia [6], developed a new control technique for handling the aerodynamic forces. The controller is composed by a Backstepping that is used to stabilize the system and also by a sliding mode which compensates the disturbances.

Bouabdallah and Siegwart [7], proposed two nonlinear control techniques and applied them to an autonomous micro helicopter called Quadrotor. A backstepping and a sliding-mode techniques. They performed various simulations in open and closed loop and also implemented several experiments on the test-bench to validate the control laws.

In this work, it is presented a control scheme that combines two controller's approaches. The combined controller approach is composed by two components. The first part is a Backstepping controller, its objective is to create a virtual control law which permits to calculate the new references for controlling roll and pitch angles. Secondly, a sliding mode controller is used, it executes control over the rotor and thus allows stabilization of the position and the appropriate monitoring of the planned trajectory to follow. The references for the sliding mode controller are provided by the Backstepping control. Then, the proposed algorithm is compared against a Backstepping-PID controller by simulations.

The combination of both schemes should produce a controller approach to strengthen and to improve the performance and robustness of both controllers. The system's performance is tested and ISE performance index used to measure it.

This article is organized as follows. Section 2 presents the dynamic model of the quadcopter. Section 3 explains the basic concepts about sliding mode and Backstepping controllers. Section 4 shows the design of the controllers. Section 5 presents the results of the simulations tests using the designed controllers. Finally, section 6 presents the conclusions of this work.

II. Dynamic Model of a Quadcopter

The quadcopter can be seen as a multivariable system having six degrees of freedom which correspond to three translational (x, y, z) and three rotational ($\emptyset,\ \theta, \psi$).

The quadcopter has four arms forming a cross, located at each end there are four actuators each one form by a motor and a propeller (two rotate clockwise and two in the opposite direction). The quadcopter has three types of motion: roll ($\emptyset$), pitch (θ) and yaw (ψ). Quadcopter position is controlled by changing the speed at which the rotors rotate.

The general sketch of a quadcopter is shown in Fig. 1.

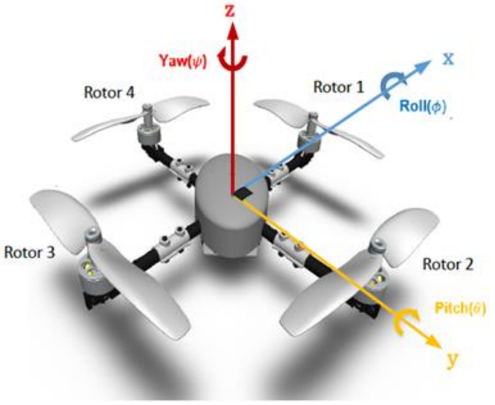

Fig. 1. Basic structure of a Quadcopter.

The generalized coordinates of the model can be written as:

$$h = [x\ y\ z\ \emptyset\ \ \theta\ \psi] \tag{1}$$

Where $(x,\ y,\ z)$ are the linear positions and $(\emptyset,\ \theta, \psi)$ are the roll, pitch and yaw angles respectively. Now, it is presented the mathematical model of the quadcopter used for this analysis:

$$\ddot{x} = \frac{(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi)}{m} U_1 \tag{2}$$

$$\ddot{y} = \frac{(\sin\psi \sin\theta \cos\phi - \sin\phi \cos\psi)}{m} U_1 \tag{3}$$

$$\ddot{z} = -g + \frac{(\cos\phi \cos\theta)}{m} U_1 \tag{4}$$

$$\ddot{\phi} = \dot{\psi}\dot{\theta}\left(\frac{I_y - I_z}{I_x}\right) + \frac{U_2}{I_x} + \dot{\psi}\dot{\theta} - \frac{J_r \dot{\theta} \Omega_r}{I_x} \tag{5}$$

$$\ddot{\theta} = \dot{\psi}\dot{\phi}\left(\frac{I_z - I_x}{I_y}\right) + \frac{U_3}{I_y} - \dot{\psi}\dot{\phi} + \frac{J_r \dot{\phi} \Omega_r}{I_y} \tag{6}$$

$$\ddot{\psi} = \dot{\theta}\dot{\phi}\left(\frac{I_x - I_y}{I_z}\right) + \frac{U_4}{I_z} + \dot{\theta}\dot{\phi} \tag{7}$$

Where, the mass of the quadrotor is represented by m, g is the gravity, Ω_r is the angular speed of all propeller, J_r is the inertia of the rotor and I_x, I_y and I_z are the inertia of the quadrotor in x, y and z respectively.

The actuators equations are presented in (8), (9), (10) y (11). The input signal U_1 is the total drag of the rotors. U_2, U_3 and U_4 are the moments for pitch, roll and yaw respectively.

$$U_1 = b\left({\Omega_1}^2 + {\Omega_2}^2 + {\Omega_3}^2 + {\Omega_4}^2\right) \tag{8}$$

$$U_2 = b\ l\left(-{\Omega_2}^2 + {\Omega_4}^2\right) \tag{9}$$

$$U_3 = b\ l\left(-{\Omega_1}^2 + {\Omega_3}^2\right) \tag{10}$$

$$U_4 = d\left(-{\Omega_1}^2 + {\Omega_2}^2 - {\Omega_3}^2 + {\Omega_4}^2\right) \tag{11}$$

Where $\Omega_1, \Omega_2, \Omega_3, \Omega_4$ the angular speed for the rotors are, b is the drag factor, d is the drag and l is the distance between the mass center and the rotor.

III. Controllers Basic Concepts

In this section, a briefly description of both controllers are presented.

A. Backstepping Control

Backstepping is a recursive Lyapunov-based scheme proposed in the beginning of 1990s [4].

The idea of this approach is to synthesize a controller recursively by considering some of the state variables as "virtual control" and designing for them intermediate control laws. The control law is synthesized to force the system to follow the desired trajectory [8].

B. Sliding Mode Control

Sliding Mode Control (SMC) is a robust control approach for nonlinear systems [9]. It faces uncertainty and nonlinearities with a better performance and robustness than classic controllers.

The sliding mode control design aims to make the system states converge to a sliding surface and then keep on it. The sliding surface $s(t)$ represents the desired dynamics of the system, which is defined [10] by:

$$s(t) = \dot{e} + \lambda e \tag{12}$$

Where λ represents a tuning parameter. The control objective is to ensure that the controlled variable is equal to the reference value, so, the error and its derivatives are zero. The problem of tracking a reference value can be reduced to that of keeping *s(t)* at zero. Once, *s(t)* is reached, it is desired to make:

$$\frac{ds(t)}{dt} = 0 \tag{13}$$

Once the sliding surface has been selected, attention must be turned to design the control law that satisfies *s(t)*=0. The control law, U(t), consists of two additive parts; a continuous part, $u_{eq}(t)$, and a discontinuous part, $u_{cr}(t)$. Therefore, the control law is given by:

$$u(t) = u_{eq}(t) + u_{cr}(t) \tag{14}$$

Where $u_{eq}(t)$ is the continuous part of the controller responsible for maintaining the controlled variable on the sliding surface and $u_{cr}(t)$ is the discontinuous part of the controller which is responsible for the state converges to the sliding surface, and it must satisfy the following inequality [11,12]:

$$s\dot{s} < 0 \tag{15}$$

The above relation means that the time derivatives of error states vector always point to the sliding surface, when the system is in reaching mode, hence the dynamics of the system will approximate the dynamic surface in a finite time.

$u_{eq}(t)$ is given by:

$$u_{eq}(t) = f(R(t), X(t)) \tag{16}$$

Where $R(t)$ is the reference signal and $X(t)$ is the model output, $u_{eq}(t)$ will be determined later using the equivalent control procedure [9] . $u_{cr}(t)$ has a non-linear element that includes the switching element of the control's law [9, 10]

$$u_{cr}(t) = K_D sign(s(t)) \tag{17}$$

To smooth the discontinuity, a sigmoid function is used [11]:

$$u_{cr}(t) = K_D \frac{s(t)}{|s(t)|+\delta} \tag{18}$$

Where K_D is a tuning parameter responsible for the speed adjustment, and δ is a parameter responsible for reducing high frequency oscillations around the desired equilibrium point, these undesirable oscillations are known as chattering [9, 11, 12].

IV. CONTROLLERS

The quadcopter has six degrees of freedom, with input signals which are responsible for making it moves forward, backward, right and left, up or down. To control the system, four equations to be connected to U_1, U_2, U_3 and U_4 were designed. U_1 defines the altitude reference and U_2, U_3 and U_4 define quadcopter roll, pitch and yaw references.

The next part presents the development of both controllers. Firstly, the Backstepping is shown and secondly, the sliding mode is treated.

A. *Backstepping Control Translation System*

Considering $u_x = \cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi$ and $u_y = \sin\psi \sin\theta \cos\phi - \sin\phi \cos\psi$ and replacing them into Eq. (2) and Eq. (3), [7]:

$$\ddot{x} = \frac{U_1}{m} u_x \tag{19}$$

$$\ddot{y} = \frac{U_1}{m} u_y \tag{20}$$

1) *Position X*

The Backstepping control procedure to design the controllers for translation system is presented in [4].

Position error X is given by:

$$e_x = x_{ref} - x \tag{21}$$

Differentiating the previous equation:

$$\dot{e}_x = \dot{x}_{ref} - \dot{x} \tag{22}$$

The goal is to design a virtual control x^* which makes $\lim_{t\to\infty} e_x \to 0$, considering the Lyapunov function V_x:

$$V_x = \frac{1}{2} {e_x}^2 > 0 \tag{23}$$

And the derivate of this function is:

$$\dot{V}_x = e_x \dot{e}_x = e_x(\dot{x}_{ref} - \dot{x}) \tag{24}$$

Introducing the virtual control:

$$x^* = \dot{x}_{ref} + q e_x \tag{25}$$

Where q is a positive constant value. Defining a new variable:

$$e_{x1} = x^* - \dot{x} = \dot{x}_{ref} + q e_x - \dot{x} \tag{26}$$

Rearranging Eq. (26), it is obtained:

$$\dot{x}_{ref} = e_{x1} - q\, e_x + \dot{x} \tag{27}$$

By replacing Eq. (27) into Eq. (24) the following result is obtained:

$$\dot{V}_x = e_x(e_{x1} - q\, e_x + \dot{x} - \dot{x}) = -q {e_x}^2 + e_x e_{x1} \tag{28}$$

The derivative of Eq. (26) is:

$$\dot{e}_{x1} = \ddot{x}_{ref} + q \dot{e}_x - \ddot{x} \tag{29}$$

Replacing Eq. (19) in Eq. (29), it is obtained:

$$\dot{e}_{x1} = \ddot{x}_{ref} + q \dot{e}_x - \frac{U_1}{m} u_x \tag{30}$$

In order to get $\lim_{t\to\infty} e_{x1} \to 0$, it is necessary to choose a V_{xx} Lyapunov control function:

$$V_{xx} = V_x + \frac{1}{2} {e_{x1}}^2 > 0 \tag{31}$$

Obtaining the derivate we have:

$$\dot{V}_{xx} = \dot{V}_x + e_{x1} \dot{e}_{x1} \tag{32}$$

Replacing equation Eq. (29) in Eq. (32), it is obtained:

$$\dot{V}_{xx} = \dot{V}_x + e_{x1}\left(\ddot{x}_{ref} + q \dot{e}_x - \frac{U_1}{m} u_x\right) \tag{33}$$

Substituting Eq. (28) in Eq. (33):

$$\dot{V}_{xx} = -q {e_x}^2 + e_x e_{x1} + e_{x1}\left(\ddot{x}_{ref} + q \dot{e}_x - \frac{U_1}{m} u_x\right) \tag{34}$$

It is proposed that:

$${u_x}^* = \frac{m}{U_1}\left[\ddot{x}_{ref} + q \dot{e}_x + e_x + p e_{x1}\right] \tag{35}$$

Assuming now perfect velocity tracking $u_x \cong {u_x}^*$, and replacing Eq. (35) in Eq. (34):

$$\dot{V}_{xx} = -q {e_x}^2 - p {e_{x1}}^2 < 0 \tag{36}$$

where $e_x \to 0$ and $e_{x1} \to 0$ with $t \to \infty$

Finally, the *x* control law is:

$${u_x}^* = \frac{m}{U_1}\left[\ddot{x}_{ref} + q \dot{e}_x + e_x + p\left(\dot{x}_{ref} + q e_x - \dot{x}\right)\right] \tag{37}$$

Desired pitch angle

Where the desired pitch angle is θ_{ref}. From Eq. (2), θ_{ref}. is obtained:

$$\theta_{ref} = sin^{-1}\left(\frac{\frac{\ddot{x}m}{U_1} - sin\psi sin\phi}{cos\psi cos\phi}\right) \quad (38)$$

Replacing in Eq. (19) in Eq. (38) and considering $u_x \cong u_x{}^*$:

$$\theta_{ref} = sin^{-1}\left(\frac{u_x{}^* - sin\psi sen\phi}{cos\psi cos\phi}\right) \quad (39)$$

2) Position Y

Following the same procedure for the *X* control, is obtained:

$$u_y{}^* = \frac{m}{U_1}\left[\ddot{y}_{ref} + q\dot{e}_y + e_y + p\left(\dot{y}_{ref} + qe_y - \dot{y}\right)\right] \quad (40)$$

Desired roll angle

Where the desired roll angle is ϕ_{ref}. Substituting Eq. (2) into Eq. (3), ϕ_{ref} is obtained:

$$\phi_{ref} = sin^{-1}\left(sin\psi\frac{\ddot{x}m}{U_1} - cos\psi\frac{\ddot{y}m}{U_1}\right) \quad (41)$$

Replacing Eq. (19) and Eq. (20), it is obtained:

$$\phi_{ref} = sin^{-1}\left(sin\psi\, u_x{}^* - cos\psi\, u_y{}^*\right) \quad (42)$$

B. *Sliding Mode Control (SMC) for Altitude and Rotational Systems*

1) Altitude Control law

The sliding surface is defined in Eq. (12) and Eq. (15). The error is defined by the difference between the desired altitude and the quadcopter altitude:

$$e_z = z_{ref} - z \quad (43)$$

Differentiating the previous equation:

$$\dot{e}_z = \dot{z}_{ref} - \dot{z} \quad (44)$$

And substituting Eq. (43) and Eq. (44) in Eq. (12):

$$s = \left(\dot{z}_{ref} - \dot{z}\right) + \lambda\left(z_{ref} - z\right) \quad (45)$$

The derivative of the previous equations, is given by:

$$\dot{s} = \left(\ddot{z}_{ref} - \ddot{z}\right) + \lambda\left(\dot{z}_{ref} - \dot{z}\right) \quad (46)$$

Using Eq. (4) and replacing in Eq. (46):

$$\dot{s} = \left(\ddot{z}_{ref} + g - \frac{\cos\phi\cos\theta}{m}U_1\right) + \lambda\left(\dot{z}_{ref} - \dot{z}\right) \quad (47)$$

Assuming now perfect velocity tracking $u(t) \cong U_1$, and using Eq. (14), and replacing $u(t)$ in U_1:

$$\dot{s} = \left(\ddot{z}_{ref} + g - \frac{\cos\phi\cos\theta}{m}\left(u_{eq} + u_{cr}\right)\right) + \lambda\left(\dot{z}_{ref} - \dot{z}\right) \quad (48)$$

To find u_{eq}, it is assumed $u_{cr} = 0$ and $\dot{s} = 0$, then:

$$u_{eq} = \frac{m}{\cos\phi\cos\theta}\left[\ddot{z}_{ref} + g + \lambda\left(\dot{z}_{ref} - \dot{z}\right)\right] \quad (49)$$

To find u_{cr}, a Lyapunov candidate function is defined by:

$$V = \frac{1}{2}s^2 > 0 \quad (50)$$

And its derivate is:

$$\dot{V} = s\dot{s} < 0 \quad (51)$$

Replacing Eq. (48) into Eq. (51)

$$\dot{V} = s\left(\left(\ddot{z}_{ref} + g - \frac{\cos\phi\cos\theta}{m}\left(u_{eq} + u_{cr}\right)\right) + \lambda\left(\dot{z}_{ref} - \dot{z}\right)\right) < 0 \quad (52)$$

Substituting Eq. (49) in Eq. (52)

$$\dot{V} = s\left(-\frac{\cos\phi\cos\theta}{m}u_{cr}\right) = -k_{cr}\, s\, u_{cr} < 0 \quad (53)$$

Where $k_{cr} = \frac{\cos\phi\cos\theta}{m}$, the ϕ and θ angles are limited to $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ [13], with this ensures $k_{cr} > 0$.

Rearranging Eq. (53) and for that $\dot{V} < 0$, u_{cr} is defined as:

$$u_{cr} = k_{cr}\, k_{dr} sign(s)\ ;\ k_{dr} > 0 \quad (54)$$

To facilitate implementation, it is considered $K_D = k_{cr}k_{dr} > 0$.

Adding Eq. (49) and Eq. (54), and $u(t) \cong U_1$ becomes:

$$U_1 = \frac{m}{\cos\phi\cos\theta}\left[g + \ddot{z}_{ref} + \lambda\left(\dot{z}_{ref} - \dot{z}\right)\right] + K_D sign(s) \quad (55)$$

Finally, the altitude control law is:

$$U_1 = \frac{m}{\cos\phi\cos\theta}\left[g + \ddot{z}_{ref} + \lambda\left(\dot{z}_{ref} - \dot{z}\right)\right] + K_D\frac{s(t)}{|s(t)|+\delta} \quad (56)$$

2) Rotational Control law

Following the same procedure for the altitude control, is obtained the controllers for roll, pitch and yaw:

$$U_2 = I_x\left[\ddot{\phi}_{ref} - \dot{\psi}\dot{\theta}\left(\frac{I_y - I_z}{I_x}\right) - \dot{\psi}\dot{\theta} + \frac{J_r\dot{\theta}\Omega_r}{I_x} + \lambda\left(\dot{\phi}_{ref} - \dot{\phi}\right)\right] + K_D\frac{s(t)}{|s(t)|+\delta} \quad (57)$$

$$U_3 = I_y\left[\ddot{\theta}_{ref} - \dot{\psi}\dot{\phi}\left(\frac{I_z - I_x}{I_y}\right) + \dot{\psi}\dot{\phi} - \frac{J_r\dot{\phi}\Omega_r}{I_y} + \lambda\left(\dot{\theta}_{ref} - \dot{\theta}\right)\right] + K_D\frac{s(t)}{|s(t)|+\delta} \quad (58)$$

$$U_4 = I_z\left[\ddot{\psi}_{ref} - \dot{\theta}\dot{\phi}\left(\frac{I_x - I_y}{I_z}\right) - \dot{\theta}\dot{\phi} + \lambda(\dot{\psi}_d - \dot{\psi})\right] + K_D\frac{s(t)}{|s(t)|+\delta} \quad (59)$$

V. SIMULATION RESULTS

In this section, the servo and regulation tasks for the quadrotor, for the circular trajectory, are presented, the quadcopter parameters are presented by [14]. The controllers were calibrated manually by trial and error. The tuning parameters for the SMC with Backstepping and the PID tuning parameters [15] with Backstepping controllers [4] are shown in the Table I and Table II:

TABLE I
SMC AND PID TUNING PARAMETERS

	SMC			PID		
Parameter	λ	δ	k_D	k_p	k_d	k_i
z	7	0.25	50	65	10	14
ϕ	3	0.5	3.5	85	10	5
θ	3	0.5	3.5	85	10	5
ψ	5	0.2	5	65	10	10

TABLE II
BACKSTEPPING CONTROL TUNING PARAMETERS FOR SMC AND PID

	SMC		PID	
Parameter	p	q	p	q
x	2	10	5	2
y	2	10	5	2

A. *Comparison SMC vs PID*

In order to evaluate the performance of the SMC controller, it is compared against a PID controller.

Fig. 2 depicts the results for both controllers for a circular trajectory. ISE [15, 16] is used to measure the performance of both controllers. In Table III are shown the ISE and $\Delta\%$ for circular trajectory

$\Delta\%$ is defined as follows:

$$\Delta\% = \left|\frac{\mathrm{ISE}_1 - \mathrm{ISE}_2}{\left(\frac{\mathrm{ISE}_1 + \mathrm{ISE}_2}{2}\right)}\right| * 100 \quad (60)$$

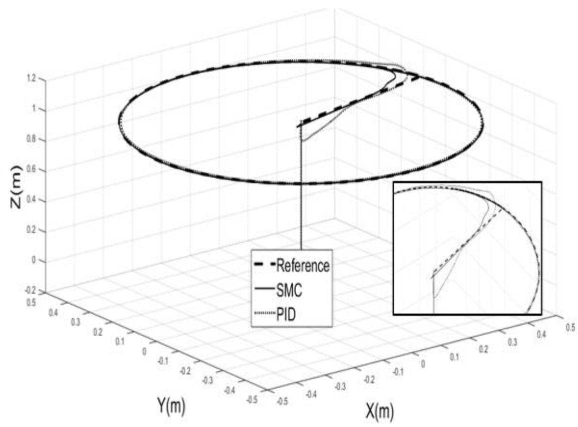

Fig. 2. Circular Trajectory without disturbances.

TABLE III
ISE FOR CIRCULAR TRAJECTORY

Parameter (ISE)	SMC	PID	Δ%
x	0,63	0,84	29,15
y	0,69	0,86	22,30
z	0,30	0,34	13,08

The results for a circular trajectory with disturbances are shown in Fig. 3

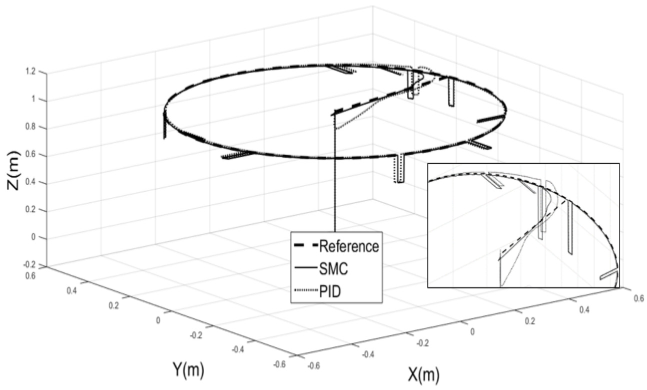

Fig. 3. Circular Trajectory with external perturbations.

B. Robustness Test

In this part robustness tests are presented. Some changes in mass are considered

Fig. 4 and Fig. 5 show robustness charts. They are presented as 3D figures where the axes are ISE index, mass changes and time. The results showed that the Backstepping-PID controller works properly until a 53.84% of additional nominal mass (0.52 Kg), while the Backstepping-SMC works properly with a 71.15% above of nominal mass (0.52 Kg), showing a be more robust.

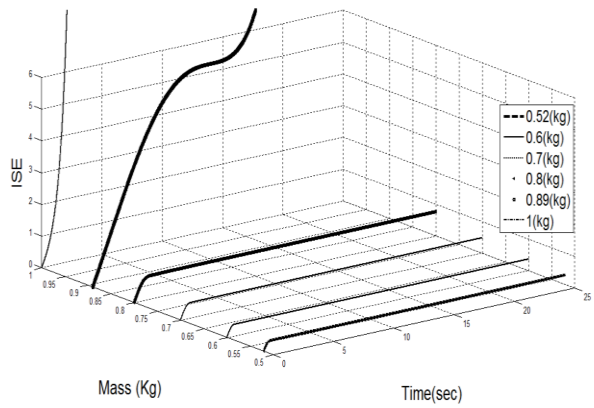

Fig. 4. Robustness chart for Backstepping- PID.

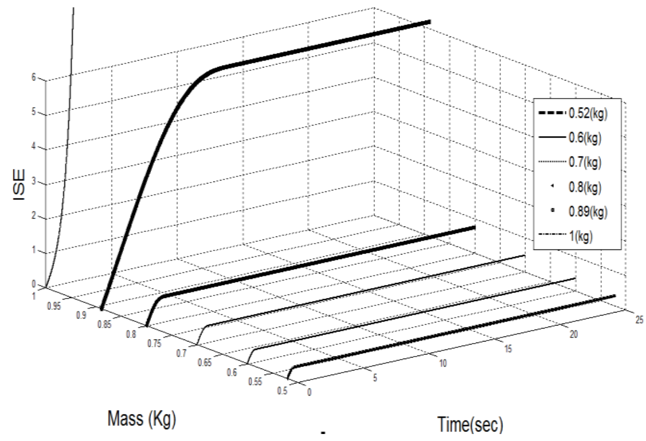

Fig. 5. Robustness chart for Backstepping- SMC.

VI. . CONCLUSIONS

A Backstepping-Sliding mode control approach was used in this paper to monitor and adjust the height and angle for a quadcopter for positions (x and y).

The results obtained by comparing Backstepping-PID and Backstepping-SMC, have shown that the second control approach presented better performance and also more robustness.

The adjustment parameters of the various controllers were performed by trial and error; it is recommended to develop tuning equations in order to improve the operation of the controllers.

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REFERENCES

[1] M. Herrera, W. Chamorro, A. Gómez, and O. Camacho, "Sliding Mode Control: An approach to Control a Quadrotor," *IEEE-APCASE, 2015*, pp. 3-4.

[2] H. Boudai, H. Bouchoucha, and M. Tadjine, "Sliding Mode Control based on Backstepping Approach for an UAV Type-Quadrotor," World Academy of Science, *Engineering and Technology International Journal of Mechanical, Aerospace, Industrial and Mechatronics Engineering*., vol. 1, pp. 39–44, 2007.

[3] S. Bouabdallah. "Design and control of quad rotors with application to autonomous flying". Ph.D. Thesis. École Polytechnique Federale de Lausanne. 2007.

[4] C. Arellano-Muro, L. Vega, B. Castillo, and A. Loukianov, "Backstepping Control with Sliding Mode Estimation for a Hexacopter," *10th Internacional Conference on Electrical, Engineering, Computing Science and Automatic Control(CCE),2013. Mexico, Mexico.* pp. 31-36.

[5] A.Kacimi, B. Mokhtari, and A. Kouadri, "Sliding Mode Control Based on Adaptive Backstepping Approch for a Quadrotor Unmanned Aerial Vehicle," *Przeglad Elektrotechniczny (Electrical Review*), pp. 188-193, 2012.

[6] H. Ramirez, V. Parra, A. Sánchez, and O. Garcia, "Robust Backstepping Control Based on Integral Sliding Models for Tracking of Quadrotors," *Journal of Intelligent & Robotic Systems.*, vol. 73, pp. 51–66, 2013.

[7] S. Bouabdallah and R. Siegwart, "Backstepping and Sliding-mode Techniques Applied to an Indoor Micro Quadrotor," *International Conference on Robotics and Automation*, 2005, Barcelona, Spain, April, pp.2259-2264.

[8] Swarup, and Sudhir, "Comparison of Quadrotor Performance Using Backstepping and Sliding Mode Control," *Proceedings of the 2014 International Conference on Circuits, Systems and Control*, 2014, Interlaken, Switzerland pp. 79-82.

[9] V.I. Utkin, "Variable Structure Systems with Sliding Mode," *IEEE Transactions on Automatic Control*, AC-22, pp. 212-222.1977.

[10] JJ. Slotine, W. Li, Applied Nonlinear Control, Prentice-Hall, New Jersey,1991.

[11] O. Camacho, C. Smith and E. Chacón. "Toward an Implementation of Sliding Mode Control to Chemical Processes". IEEE-ISIE´97. Guimaraes, Portugal, 1997. pp. 1101-1105.

[12] O. Camacho and C. Smith, "Sliding Mode Control: An approach to to regulate nonlinear chemical processes," *ISA Transaction 39*, pp. 205-218. *2000*

[13] T. Madani, and A. Benallegue, "Backstepping Control with Exact 2-Sliding Mode Estimation for a Quadrotor Unmanned Aerial Vehicle*," IEEE/RSJ International Conference on Intelligent Robots and Systems, 2007,* San Diego, USA, pp.141-146.

[14] A. Gómez, "Real Time Robustness Analysis of Quadrotor UAV´s," M. Eng. thesis, University of Glasgow, United Kingdom. 2007.

[15] T. Marlin, *Process Control: Designing Processes and Control Systems for Dynamic Performance*, Chapter 9, Mc Graw-Hill, New York, 1995

[16] E. Iglesias, J. Garcia, O. Camacho. S. Calderón, and A. Rosales, "Ecuaciones de Sintonización para Controlador por Modos Deslizantes y Control de Matriz Dinámica a partir de un Módulo Difuso," *Revista AXIOMA.*, vol. 1, pp. 14-24,2015.