Least Squares and LS-SVM LPV Identification Methods with Application on a Generating Unit

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***Abstract*—In this paper were evaluated identification methods for linear parameter-varying systems (LPV) based on least squares (LS) and support vector machines (LS-SVM). Both strategies are compared by using a collected data set from a real 350MVA generating unit. In this application is used a local approach identification of LPV models in regression forms and it is compared to a local LTI model in order to show the advantages in modelling nonlinear systems by using an LPV representation.**

***Resumen*— En este trabajo se evalúan los métodos de identificación de sistemas de parámetros lineales (LPV) sobre la base de los mínimos cuadrados (LS) y máquinas de vectores de soporte (SVM-LS). Ambas estrategias son comparados usando un conjunto de datos recogidos con ayuda de una verdadera unidad de generación de 350 MVA. En esta aplicación se utiliza un enfoque local identificación de modelos de LPV en formas de regresión y se compara con un modelo local LTI con el fin de mostrar las ventajas en el modelado de sistemas no lineales con una representación LPV.**

I. Introduction

Linear Varying Parameter (LPV) Systems are described by linear differential equations which depends on time varying parameters [1].Those parameters are known as scheduling variables or scheduling parameters, which are often exogenous and govern the dynamical behaviour of the system. The scheduling variables define operating points of the system while the relation between the system signals remain linear. Therefore, an LPV system can be viewed as a collection of linear time invariant (LTI) systems. However, it is important to note that the scheduling variables also define the dynamical behavior of the system. Hence, an LPV system is more than a simple collection of LTI systems [2].

The basic idea of the identification of an LPV system is to define the parameter dependence in terms of adequate basis function known a priori. The first papers about LPV identification assumed a priori knowledge of the basis functions and dealt exclusively on the estimation problem, normally solved in a lest-squares sense [3]. Those papers used a linear in parameters regression to explain the system behavior and the problem formulation was solved by a prediction error setting.

Assuming the knowledge of the structure of an LPV system is sometimes valid because an LPV model can be obtained directly by the differential equations which describe the system. However, in most cases this is not possible, it can be a difficult task and in many cases will need specialist knowledge. For that reason many papers on LPV identification are focusing on a semi-parametric framework through least-squares supporting vector machines (LS-SVM) [4], [5], [6]. Many papers had introduced the LS-SVM Identification framework, one of them to a class of non linear models [7] and to an Input-Output LPV (LPV-IO) representation [8], [9].

Support Vector Machines (SVM) are supervised learning tools formulated under the modern statistical learning theory originally introduced by Valpnik to solve classification problems. [10], [11]. LS-SVM is one reformulation of the SVM original method when it's considered a loss function to solve a set of linear equations instead of solving a quadratic programming problem. This method is attractive due to the convexity of the problem leading to a unique optimal solution [10].

In this paper, LPV identification strategies [1], [8], [9] are assessed with data sets previously obtained in works [12] and [13], which presented the design of adaptive power system stabilizers to the damping of electromechanical oscillations. Field tests were carried out in a 350 MVA hydro generator at Tucuruí Power Plant (north region of Brazil) and input-output data were acquired for some operating conditions of reactive power. In [12] a classical gain-scheduling controller was designed based in a local controller network and experimental control tests showed the improvement of the response when compared with local controllers.

Besides the gain-scheduling controller, results of design and field tests of an LPV damping controller were published in [13]. The LPV model was identified using the LPV LMS method showed in [3]. The objective of this paper is the evaluation of LPV-IO models to represent the non-linear dynamics of the hydro generator, instead of several local models as in [12]. Two LPV identification methods are evaluated, a classical LPV least squares [1] and the LS-SVM approach presented in [8], [9].

It is worthy to remark that performing on-line identification tests in large electrical generating units is a longstanding and very important scientific and technological subject, as these systems present time-variable and nonlinear behavior due to varying loading condition or unforeseeable events, such as changes in the line transmission configuration. Furthermore, a generating unit is a critical equipment regarding its demand for remaining in continuous operation and is subject to very restrictive safety and regulation rules. As a consequence, it's not so easy to grant permission for performing identification field tests in such a system. Therefore, it is of paramount importance to propose new identification techniques able of capturing the relevant system's dynamics, for a whole set of operating conditions. Therefore, it is a major contribution of this paper to propose a LPV identification methodology able to capture the operating point variable system dynamics without the need of interrupting the normal system operation.

II. LPV System Identification

There are many LPV system representations available, such as, LPV-IO, LPV State Space (LPV-SS) representation, Linear Fractional Transformation (LFT) description, and Orthonormal Basis Function (OBF) [2]. In this paper is considered an input-output representation. In the discrete time, one of the most common model structures that relates input and output in system identification is the autoregressive with exogenous input (ARX) model [14]. This model is extended to an LPV-ARX SISO (Single Input Single Output) model in (1).

$$y(k)+\sum_{i=1}^{n_a} a_i(p_k)y(k-i) = \sum_{j=1}^{n_b} b_j(p_k)u(k-j-n_k)+e(k) \quad (1)$$

where k is the discrete time, u and y are, respectively, the input and output, $p_k = p(k)$ is the scheduling variable, n_k is the discrete delay, and e is white noise with zero mean and standard deviation σ^2. The coefficients (a_i,b_j) of the LPV-ARX model depend statically on p.

To be able to identify an LPV model in regression form it is necessary to define a set of basis function (ψ_{i_j}) to parametrize the coefficients as

$$a_i(p(k)) = \theta_{i0} + \theta_{i1}\psi_{i1}(p(k)) + \dots \theta_{is_i}\psi_{is_i}(p(k)), \quad (2)$$

where θ_{ij} are the parameters to be estimated. This makes the identification problem to be in a regression form and the model to be linear in parameters, which can be solved by least squares in the prediction error framework. Additionally, this parametrization makes it possible to formulate an LPV extension to the prediction error framework, making it possible to analyse the stochastic properties of the estimation [2], [8].

Besides the order of the model structure (choice of n_a, n_b, delay), it is necessary an adequate choice of the set of basis functions $\{\psi_{ij}\}$ in order to identify the dependencies of a_i and b_i on p. This dependence could be polynomial or even rational and discontinuous functions [2].

In order to formulate the LPV identification in a regression form is defined

$$\left[\phi_1 \dots \phi_{n_g}\right]^T \equiv \left[a_1 \dots a_{n_a} b_1 \dots b_{n_b}\right]^T, \quad (3)$$

where $n_g = n_a + n_b$, and each $\phi(\cdot)$ is one function with static dependence on p. In this form the process (1) can be completely characterized by $\{\phi_i(\cdot)\}_{i=1}^{n_g}$. This form is almost exclusively done in the LPV literature [3], [8], [1]. Assuming that each ϕ_i is linear parametrized as

$$\phi_i(\cdot) = \theta_{i0} + \sum_{j=1}^{s_i} \theta_{ij}\psi_{ij}(\cdot), \quad (4)$$

where $\{\theta\}_{i=1,j=1}^{n_g,s_i}$ are the unknown parameters and $\{\psi_{ij}\}_{i=1,j=1}^{n_g,s_i}$, with $s_i \in \mathbb{N}$ are functions chosen by he user. In this case, (1) can be written in regression form as

$$y(k) = \theta^T\varphi(k) + e(k), \quad (5)$$

where $\theta = \left[\theta_{1,0} \ \cdots \ \theta_{1,s_1} \ \theta_{2,0} \ \cdots \theta_{2,s_2} \ \cdots \ \theta_{n_g,0} \ \cdots \theta_{n_g,s_{n_g}}\right]^T$ and

$$\begin{aligned}\varphi(k) = [&-y(k-1) - \psi_{11}(p_k)y(k-1)\cdots \\ &-\psi_{1s_1}(p_k)y(k-1)\cdots - \psi_{n_a s_{n_a}}(p_k)y(k-n_a)\cdots \\ &u(k-n_k)\cdots - \psi_{n_g s_{n_g}}(p_k)u(k-n_b-n_k+1)]^T.\end{aligned}$$

Given a data set $D^N = \{u\ , y\ , p\}_{i=1}^N$, where N is the total number of samples in the data set. The least squares estimates of (5) is given by

$$\hat{\theta} = \arg\min V(\theta, e), \quad (6)$$

where

$$V \equiv \frac{1}{N} \parallel e(k) \parallel_{l_2}^2, \quad (7)$$

and $e(k) = y(k) - \theta^T\varphi(k)$.

In order to guarantee the identifiability and a unique solution is considered that the regressors are persistently excited.

Organizing the data in matrices as

$$Y = [y(1) \ \ y(2) \ \ \cdots \ \ y(N)]^T, \quad (8a)$$

$$\Phi = [\varphi(1) \ \ \varphi(2) \ \ \cdots \ \ \varphi(N)]^T. \quad (8b)$$

The LS solution of (6) can be obtained from (9).

$$\hat{\theta}_N = \left(\Phi^T\Phi\right)^{-1}\Phi^T Y. \quad (9)$$

III. LS-SVM for LPV Systems

In this section, it is showed how the SVM approach can be made with respect to the estimate of (1) without specifying the underlying dependencies needed for the optimization process via least squares as demonstrated in [8].

A. LPV modeling in a SVM configuration

In contrast with the form of identification presented in Section II, the structural dependency of the coefficients ϕ_i is assumed to be a priori unknown. Consequently, the parametrized model 1 may be introduced as

$$M_{\omega,\varphi} : y(k) = \sum_{i=1}^{n_g} \omega_i^T \phi_i(p_k) x_i(k) + e(k), \tag{10}$$

where each $\phi_i : \mathbb{R} \to \mathbb{R}^{n_H}$ denotes an undefined mapping, potentially infinite and $\omega_i \in \mathbb{R}^{n_H}$ is the i^{th} vector of parameters and

$$x_i(k) = y(k-i),\ i = 1, \cdots, n_a, \tag{11a}$$

$$x_{n_a+1+j}(k) = u(k-j),\ j = 0, \cdots, n_b. \tag{11b}$$

Additionally, one can write $\omega = [\omega_1^T \cdots \omega_{n_g}^T]^T \in R^{n_g n_H}$ and

$$\varphi(k) = [\phi_1^T(p_k)x_1(k) \ \ \cdots \ \ \phi_{n_g}^T x_{n_g}(k)]^T. \tag{12}$$

Thus, equation (9) can be rewritten as

$$y(k) = \omega^T \varphi(k) + e(k), \tag{13}$$

the LS-SVM approach aims minimizing the following cost function [10]

$$J(\omega, e) = \frac{1}{2} \sum_{i=1}^{ng} \omega_i^T \omega_i + \frac{\gamma}{2} \sum_{k=1}^{N} e^2(k), \tag{14}$$

where $\gamma \in \mathbb{R}$ is a factor of regularization. The equation (14) is the sum-of-norms criterion, containing both the error term from the equation (10) as the Euclidean norm term of the parameter vector ω [9]. Consider $M_{\omega,\varphi}$ model proposed in (10) whose estimation corresponds to the following optimization problem

$$\min J(\omega, e) = \frac{1}{2} \sum_{i=1}^{ng} \omega_i^T \omega_i + \frac{\gamma}{2} \sum_{k=1}^{N} e^2(k), \tag{15a}$$

$$e(k) = y(k) - \sum_{i=1}^{n_g} \omega_i^T \phi_i(k) x_i(k). \tag{15b}$$

This constrained optimization problem can be solved by the Lagrange multipliers in the dual space [10], [8]. The Lagrangian can be defined as

$$\begin{aligned} L(\omega, e, \alpha) = J(\omega, e) - \\ \sum_{k=1}^{N} \alpha_k \left(\sum_{i=1}^{n_g} \omega_i^T \phi_i(k) x_i(k) + e(k) - y(k) \right), \end{aligned} \tag{16}$$

with $\alpha_k \in \mathbb{R}$ being the Lagrange multipliers. The global optimum can be achieved when

$$\frac{\partial L}{\partial e} = 0 \to \alpha_k = \gamma e(k), \tag{17a}$$

$$\frac{\partial L}{\partial \omega_i} = 0 \to \omega_i = \sum_{k=1}^{N} \alpha_k \phi_i(k) x_i(k), \tag{17b}$$

$$\frac{\partial L}{\partial \alpha_k} = 0 \to e(k) = y(k) - \sum_{i=1}^{n_g} \omega_i^T \phi_i(k) x_i(k), \tag{17c}$$

replacing 17a and 17b into 17c, the following set of equations is obtained

$$y(k) = \sum_{i=1}^{n_g} \left(\sum_{k=1}^{N} \alpha_k x_i(k) \phi_i^T(k) \right) \phi_i(k) x_i(k) + \gamma^{-1} \alpha_k \tag{18}$$

for $k \in \{1, ..., N\}$. This is equivalent to

$$Y = (\Omega + \gamma^{-1} I_N)\alpha, \tag{19}$$

where $\alpha = [\alpha_1 \ \ \cdots \ \ \alpha_N]^T \in \mathbb{R}^N$, and Ω is called the kernel matrix, which is defined as

$$[\Omega]_{j,k} = \sum_{i=1}^{n_g} [\Omega^i]_{j,k}, \tag{20}$$

with

$$\begin{aligned} [\Omega^i]_{j,k} &= x_i(j) \phi_i^T(j) \phi_i(k) x_i(k) \\ &= x_i(j) \langle\rangle \phi_i(j), \phi_i(k) \rangle x_i(k) \\ &= x_i(j) (K^i(p(j), p(k))) x_i(k). \end{aligned}$$

Here K^i is a positive definite function that defines the inner product $\phi_i^T(j)\phi_i(k)$. Consequently, K^i define Ω, characterizing the mapping $\{\phi_i\}_{i=1}^{n_g}$. This allows the characterization of a wide range of nonlinear dependencies as a linear combination of infinite functions defined by ($n_H = \infty$) and the choice of particular inner product. Called the Kernel trick [11], [15] , this approach enables the identification of the coefficients a_i and b_j without explicitly defining the feature maps involved. This same trick is used in many other optimization problems such as pattern classification of nonlinear separability data. A typical kernel is, for instance, the radial basis kernel (RBF) defined as

$$K^i(p_j, p_k) = \exp \left(- \frac{\|p_j - p_k\|_{l_2}^2}{\sigma_i^2} \right), \tag{21}$$

although other kernels can be used, such as, the polynomial and sigmoid kernels. The choice of kernel defines a dependency class from which the problem can be represented. Choosing a particular kernel, it is possible to obtain the solution of (19) as

$$\alpha = (\Omega + \gamma^{-1} I_N)^{-1} Y. \tag{22}$$

The solution obtained minimizes (15a-b). The model estimation can be obtained according with 17b. In this way, the estimated coefficients are obtained as

$$a_i(\cdot) = \omega_i^T \phi_i(\cdot) = \sum_{k=1}^{N} \alpha_k x_i(k) K^i(p(k), \cdot), \tag{23a}$$

$$b_j(\cdot) = \omega_{\tilde{j}}^T \phi_{\tilde{j}}(\cdot) = \sum_{k=1}^{N} \alpha_k x_{\tilde{j}}(k) K^{\tilde{j}}(p(k), \cdot), \tag{23b}$$

where $\tilde{j} = n_a + j$. Thus it is not necessary to know the vector of parameters ω to estimate the model coefficients, just the vector α and the defined kernel functions. Note that only the product $\omega_i^T \phi_i(\cdot)$ was accessible through the kernel trick. The high dimensional parameters ω and the parameter mapping ϕ_i is not directly accessible.

B. Advantages of a semi-parametric formulation

The LPV-SVM strategy does not require a specific statement of the parameter mapping ϕ_i and neither the high dimensionality estimation of vector parameters ω_i. It only requires the kernel functions K^i for $i = 1, \cdots, n_g$. Thus, one can define Kernel functions independents from each other. This allows the insertion of a priori knowledge of the system behavior.

Compared to LS strategy in Section II, the LS-SVM strategy is not easily subject to the problem of over-parametrization and does not need basis functions defined explicitly, thus the LS-SVM strategy can represent a large number of dependencies according to the choice of the kernel function.

IV. Description of the Application

The strategies presented in sections II and III were applied in a data set of a large generator system, originally collected in [12], at Tucuruí hydroeletric power plant(THPP).

Figure 1 shows the instrumentation used during the data acquisition field tests. Detailed block diagram of the generating unit can be seen in [12], [13].

Figure 1: Instrumentation equipments

THPP is a very important power plant of the Brazilian interconnected power system (IPS). The power plant is made up of a set of 23 generating units (12 350 MVA and 11 375 MVA generating units), adding up to a total maximum generation capacity of 8325 MVA.

The synchronous machines are equipped with fast thyristor-based excitation systems along with high-gain AVR. Each generating unit is equipped with a fixed-parameter PSS tuned to damp inter-area electromechanical modes observed in the large IPS.

To perform the identification procedure the system was excited with a pseudo random binary signal (PRBS) of low-amplitude to the AVR controller of a 350MVA generating unit. The PRBS signal was designed to excite the range of frequencies of the electromechanical oscillation modes in electric power systems. The PRBS sequence was generated in order to excite uniformly the system modes in the band 0.02–5.5 Hz, which contains the natural frequency of the dominant electromechanical oscillation mode observed in the generating unit [12].

The proposed dataset follows a local approach of LPV system identification. The plant was excited while the level of reactive power of the generator machine (scheduling variable) was maintained constant. The data set contains three different operation points defined by the level of the reactive power. Figure 2 illustrates (part of the data set) the input (u - PRBS signal on the AVR), output(y - active power deviation) of the system and the scheduling variable (p - reactive power).

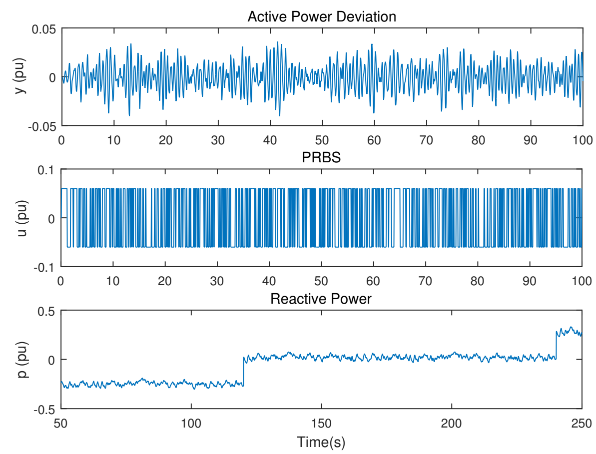

Figure 2: Data from the generating unit

V. Results

The dataset in Figure 2 was divided in two datasets, one for the estimation and the other to validate the model. The estimation dataset D_E is composed by equally distributing 80% of the total original data set, while the remaining data is used to compose the validation dataset D_V.

The two strategies of sections II and III were applied to the estimation data set D_E. In both strategies it is investigated the orders of the system by varying n_a, n_b and n_k.

For the LS strategy is used a polynomial dependence on p such as (4) in each $\psi_{ij} = p^j$ with $j = 1, \cdots n_p$, this formulation is very common on LPV system identification literature [3]. While in the LS-SVM framework is adopted a RBF kernel. Many system structures were tested ($n_s = 1, \cdots, 10$, with $s = \{a, b, k, p\}$). It was noticed that the system could be well represented by a model of order $n_a = n_b = 4$ without discrete delay $n_k = 1$. High order models performed better than fourth order, but increasing the order of the model can over-parametrize and the performance showed it is not much better than the model($na = 4$, $nb = 4$, $nk = 1$). Moreover, the fourth order model presented in [12] was demonstrated to achieve good performances for control purposes.

To analyse the quality of the models obtained is used a fitness function called best fir ratio (BRF) which is largely used in the system identification to indicate the performance of the model in relation to validation data [14], [8], [9], [16]. The BFR is defined as

$$BFR = 100\% max\left(1 - \frac{\|x(k) - \hat{x}(k)\|_{l_2}}{\|x(k) - \bar{x}\|_{l_2}}, 0\right), \quad (24)$$

where $\bar{x}$ is the mean of (x). The BFR indicate how much the simulated output of the model deviates from the expected. When the BFR is close to 100, the better the model explains the data, and the closer to zero the model fails to explain the system behavior.

Another function of performance used in the LPV literature is the FID function defined as

$$FID = 100\% \max\left(1 - \frac{var(x - \hat{x})}{var(x)}, 0\right), \quad (25)$$

the idea is the same as (24), but this time is analysed the error variation in relation to the variation of the expected signal.

To illustrate the advantage of an LPV representation were estimated ARX-LTI models using the LS prediction error approach. The ARX model is estimated using only data from one operating point of D_E where the reactive power is close to zero. The estimated ARX model obtained is

$$\theta_{LS-ARX} = [-2.26 \quad 1.99 \quad -0.73 \quad 0.12 \quad -5.7 * 10^{-3} \\ -0.14 \quad 0.02 \quad 0.16]^T.$$

Table I shows the estimated parameters from the LPV-ARX via LS. Figure 3 illustrate the variation over p of the coefficients ($A(p)$ and $B(p)$) obtained in the model LPV-ARX estimated using the LS algorithm. The same illustration is done by the model obtained via LS-SVM. Figure 4 shows the variation over p of the LPV-ARX model estimated via LS-SVM.

Table I: Parameters of the estimated LPV-ARX

	p^0	p^1	p^2		p^0	p^1	p^2
$\boldsymbol{a_1}$	-2.24	0.02	-0.10	$\boldsymbol{b_1}$	-0.005	-0.006	-0.005
$\boldsymbol{a_2}$	1.97	0.04	0.01	$\boldsymbol{b_2}$	-0.14	0.07	-0.03
$\boldsymbol{a_3}$	-0.69	-0.03	0.18	$\boldsymbol{b_3}$	0.01	0.017	0.04
$\boldsymbol{a_4}$	0.11	-0.007	-0.16	$\boldsymbol{b_4}$	0.17	-0.06	0.003

Table II presents the results of the three models obtained by the algorithms presented in this paper. The results are all evaluated using D_V. The validation data set D_V is separated in three different regions (a, b and c) according with the operation point defined by reactive power. It is valid to remind that the model structure(4,4,1) represents (n_a, n_b, n_k).

As expected, the ARX-LTI model has a good performance in the region where the model is estimated, while in others regions the model barely represents the system behavior. However, the LPV-ARX models can considerably represent the system with approximately the same results in the three regions defined.

Although the strategy presented in [9] takes into account the noise in the scheduling variable, in this paper it has not been observed a great improvement in relation to the original LS-SVM introduced in [8].

Other kernels were tested, such as, polynomial and sigmoid kernels, but both were not able to overcome the Kernel RBF with respect to each evaluation function (BFR and FID).

The parameters of the kernel functions were evaluated through trial and error strategy. It has not been observed any relative improvement while tuning the RBF parameters. The σ was chosen 1.2 for $i = 1, \cdots, n_g$ and $\gamma = 1e^5$.

Table III shows the mean and the standard deviation of the error of the three models estimated in this paper.

Table III: Mean and Standard Deviation of the error

Model(4,4,1)	mean	std
ARX-LTI	-0.0047	0.0544
LPV-ARX	-0.0049	0.0161
LS-SVM-ARX	-0.0048	0.0264

VI. Conclusion

In this paper have been compared two strategies to obtain LPV-ARX models. An LTI ARX Identification was performed to illustrate the advantages of LPV models to represent a system in many operating conditions.

Both strategies had a good performance on identifying the LPV structure. The LS-SVM was not able to perform better results even considering the noisy scheduling variable case. In fact, the noisy case performs slightly better than the original LS-SVM, but not enough to overcome the least squares solution.

The best kernel function was the RBF even varying the orders of the model structure (n_a, n_b, n_k) the RBF kernel maintained a better performance related to the other kernels analysed (polynomial and sigmoid), but considerably worse than an equivalent LPV model with same order estimated by LS.

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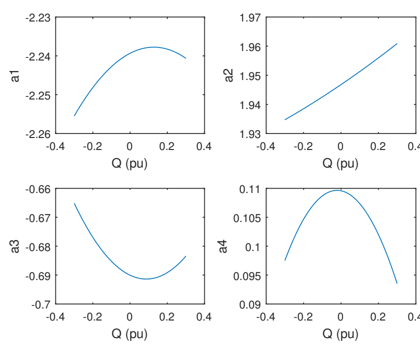
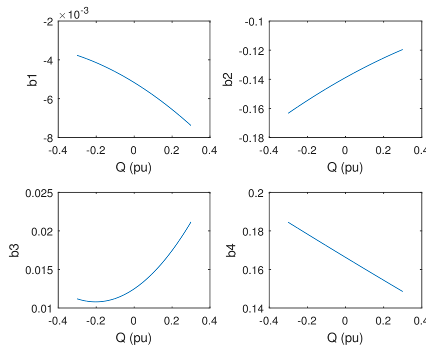
Table II: Results of the Identified models with respect to the performance functions

Model Structure(4,4,1)	BFR (a)	BFR(b)	BFR(c)	BFR global	FID (a)	FID(b)	FID(c)	FID global
ARX-LTI	52.67%	87.79%	51.59%	64.01%	76.29%	98.49%	77.81%	84.20%
LPV-ARX ($n_p = 3$)	87.70%	87.74%	88.27%	87.90%	98.62%	98.51%	98.66%	98.60%
LS-SVM LPV-ARX	87.64%	88.11%	87.12%	87.62%	98.56%	98.61%	98.39%	98.52%

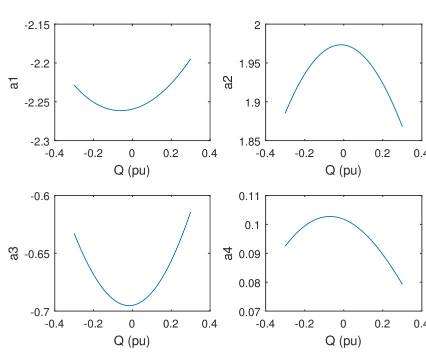


(a) Coefficientes of $A(p)$

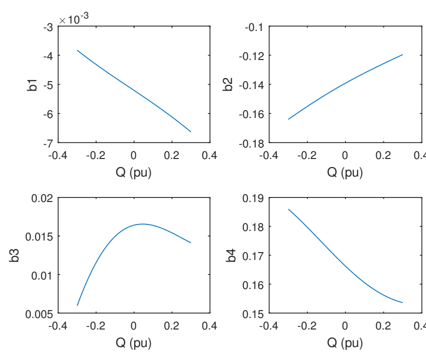


(b) Coefficientes of $B(p)$

Figure 3: Coefficients of the LPV-ARX model estimated via LS



(a) Coefficientes of $A(p)$



(b) Coefficientes of $B(p)$

Figure 4: Coefficients of the LPV-ARX model estimated via LS-SVM

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